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Orientifold theory dynamics and symmetry breaking

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We show that it is possible to construct explicit models of electroweak symmetry breaking in which the number of techniflavors needed to enter the conformal phase of the theory is small and weakly dependent on the number of technicolors. Surprisingly, the minimal model with *just* two (techni)flavors, together with a suitable gauge dynamics, can be made almost conformal. The theories we consider are generalizations of orientifold type gauge theories, in which the fermions are in either two index symmetric or antisymmetric representation of the gauge group, as the underlying dynamics responsible for the spontaneous breaking of the electroweak symmetry. We first study their phase diagram, and use the fact that specific sectors of these theories can be mapped into supersymmetric Yang-Mills theory to strengthen our results. This correspondence allows us also to have information on part of the nonperturbative spectrum. We propose and investigate some explicit models while briefly exploring relevant phenomenological consequences. Our theories not only can be tested at the next collider experiments but, due to their simple structure, can also be studied via current lattice simulations.

Dynamical breaking of the electroweak symmetry due to underlying strong gauge dynamics is a natural possibility [1], which has been intensively investigated. For a nice summary and references, see [2]. The goal of this paper is to show that it is possible to construct explicit models of electroweak symmetry breaking in which the number of techniflavors needed to enter the conformal phase of the theory is small and weakly dependent on the number of technicolors. Surprisingly, the minimal model with just two (techni)flavors, together with a suitable gauge dynamics, can be made almost conformal.

The starting point and motivation for this study is the recent argument [3] that *non*-supersymmetric Yang-Mills theories with a Dirac fermion either in the two index symmetric or antisymmetric representation of the gauge group are nonperturbatively equivalent to supersymmetric Yang-Mills (SYM) theory at large N, so that exact results established in SYM theory should hold also in these "orientifold" theories. The orientifold theories at finite N were studied in [4]. Interestingly, the two-index antisymmetric representation for $N = 3$ matches ordinary QCD. This observation was made long ago by Corrigan and Ramond [5].

In this paper we adapt the orientifold type field theories to describe dynamical electroweak symmetry breaking. In the following, we refer to fermions in the two-index antisymmetric (symmetric) representation as *A-types* (*S-types*). By studying the conformal window of these theories we will show that S-type theories provide a natural framework for walking technicolor models with a small number of techniflavors. The idea of using higher dimensional representations of the gauge group for technicolor like theories was also proposed and investigated by Eichten and Lane [6]. However, here we will show that, thanks to the identification of some sectors of these theories with SYM, we will be able to make new relevant predictions.

We then construct simple generalizations of the basic technicolor models. We show that the minimal theory with just two S-type technifermions is already near its conformal window. We also construct the next to minimal theory with one family of Dirac fermions, but with A-type gauge interactions rather than QCD-like ones. We show that this theory is almost conformal when the underlying gauge theory is $SU(4)$.

Thanks to the fact that certain sectors of these theories can be mapped into supersymmetric Yang-Mills theory our prediction for the phase diagram of the two index symmetric theories is much more robust than for ordinary non-supersymmetric theories. Besides, part of the nonperturbative spectrum of these theories is known in an expansion in the inverse of the number of colors. This allows us to suggest new phenomenological signatures for these theories. We also comment on electroweak precision measurements.

Consider N_f Dirac flavors in the two-index symmetric representation (S-types) of the $SU(N)$ gauge group. To make the phenomenologically appealing properties of the theory explicit, consider the beta-function $\beta = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2} + ...$, with

$$\beta_0 = \frac{11}{3}N - \frac{2}{3}N_f\,(N \pm 2)\,, \tag{1}$$

$$\beta_1 = \frac{34}{3}N^2 - N_f\,(N \pm 2)\left[\frac{10}{3}N + \frac{2}{N}\,(N \mp 1)\,(N \pm 2)\right]\,,$$

where the upper sign is for the two-index symmetric representation, while the lower sign is for the two-index antisymmetric representation. Note first that at infinite N

and with one Dirac flavor we recover the super Yang-Mills beta function, and for a generic N_f we recover the beta function of the theory with N_f adjoint Weyl fermions. The theory is asymptotically free for $N_f < \frac{11}{2}N/(N \pm 2)$. At infinite N A-types and S-types are indistinguishable and N_f must be < 5 to have an asymptotically free theory [3]. This result is already very interesting for phenomenology since it severely limits the number of possible techniflavors. This is very different from the technicolor theories with fermions in the fundamental representation of the gauge group. In such a case the maximum allowed number of flavors increases linearly with the number of colors: the maximum of N_f is equal to $11N/2$ for these theories.

At a finite number of colors, S-types and A-types are distinguishable from each other. For S-types asymptotic freedom is lost already for three flavors when $N = 2$ or when $N = 3$, while the upper bound of $N_f = 5$ is reached for $N = 20$ and it does not change when N is further increased. In the theory with S-types, we therefore now know that the number of flavors must be smaller than 5 for the theory to yield chiral symmetry breaking. This takes into account that there is also a conformal window of size $N_f^c < N_f < 5$, with the critical value N_f^c to be determined shortly. We will show that a theory with two S-types is very close to the conformal window from $N = 2$ and up to a quite large N. For A-types the situation is quite opposite.

Some of the problems of the simplest technicolor models, such as providing ordinary fermions with a mass, are alleviated when considering new gauge dynamics in which the coupling does not run with the scale but rather walks, i.e. evolves very slowly [7, 8, 9, 10, 11]. Achieving a slowly evolving coupling constant usually requires a quite large number of fermions in the fundamental representation of the gauge group, and the number of fermions is expected to increase linearly with the number of colors. The lowest number of fermions is obtained for two colors, and the predicted number of flavors is around eight. However, the **S**-parameter remains still a problem even if one considers nonperturbative corrections [12, 13].

We will now show how to achieve walking in the theories considered here with a low number of flavors. The critical value of flavors, N_f^c, for the transition may be estimated by making use of a perturbative expansion of the anomalous dimension of the quark mass operator γ. At the first order in perturbation theory, γ is given by: $\gamma = a_0\,\alpha$, with $a_0 = (3C_2\,[R])/2\pi$ and $C_2\left[\square\!\square\,,\begin{smallmatrix}\square\\[-2pt]\square\end{smallmatrix}\right] = (N \pm 2)(N \mp 1)/N$. We can now evaluate γ at the fixed point value of the coupling constant, which at two loops in the beta function expansion is: $\frac{\alpha^*}{4\pi} = -\frac{\beta_0}{\beta_1}$. In Ref. [14], it was noted that in the lowest (ladder) order, the gap equation leads to the condition $\gamma(2-\gamma) = 1$ for chiral symmetry breaking. To all orders in perturbation theory, this condition is gauge invariant

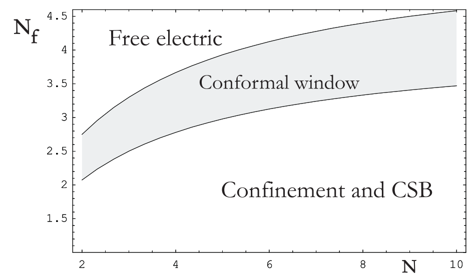

FIG. 1: Phase diagram as function of number of N_f dirac flavors and N colors for fermions in the two-index symmetric representation (S-types) of the gauge group.

(since γ is gauge invariant) and also equivalent to the condition $\gamma = 1$ of Ref. [15]. To any finite order in perturbation theory these conditions are of course different. The condition $\gamma(2-\gamma) = 1$ leads, in leading order, to the critical coupling

$$\alpha_c = \frac{\pi}{3C_2}\ , \tag{2}$$

above which the ladder gap equation has a non-vanishing solution. Using Eq. (2) in α^*, so that the condition $\gamma < 1$ is satisfied to leading order, leads to the conclusion that chiral symmetry is restored for

$$N_f > N_f^c \simeq \frac{83N^3 \pm 66N^2 - 132N}{20N^3 \pm 55N^2 \mp 60}\ . \tag{3}$$

The phase diagram as a function of the number of colors and flavors for the S-type case is presented in figure 1. We find that for $N = 2, 3, 4, 5$ $N_f = 2$ is already the highest possible number of flavors before entering the conformal window. Hence for these theories we expect a slowly evolving coupling constant. We find that $N_f \geq 3$ for $N \geq 6$ but will remain smaller than or equal to four for any N.

These estimates are based on the validity of the first few terms in the perturbative expansion of the β-function. We will provide in the next section another argument leading to the same prediction without using any of the previous arguments. The critical value of flavors increases with the number of colors for the gauge theory with S-type matter: the limiting value is 4.15 at large N. Our results are consistent with the results presented in [3] only in the infinite N limit.

The situation is different for the theory with A-type matter. As it is evident from the associated phase diagram presented in figure 2, the critical number of flavors increases when decreasing the number of colors. The maximum value of $N_f = 12$ is obtained for $N = 3$, i.e. standard QCD. We shall see that we can construct a theory of eight A-type technifermions, corresponding to the one-family model, which is already near the conformal window for $N = 4$ and hence the theory is another prototype walking technicolor model.

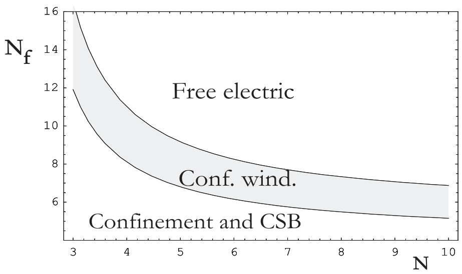

FIG. 2: Phase diagram as function of number of N_f dirac flavors and N colors for fermions in the two-index antisymmetric representation (A-types) of the gauge group.

Further constraints from super Yang-Mills

A better understanding of the nonperturbative dynamics of these theories can be obtained by exploiting their relation with supersymmetric Yang–Mills theory.

The conformal window of the S-theory provided in the previous section is more reliable than the correspondent one obtained for a generic nonsupersymmetric theroy. This is so for two reasons: 1) The S-theory looses asymptotic freedom quite soon. Already for $N_f = 3$ or $N_f = 4$ when $1 < N \leq 5$ and for $N_f = 5$ at large N. 2) The $N_f = 1$ sector of the theory (the orientifold field theory) at large N is mapped in SYM which is known to confine. Hence we can exclude the $N_f = 1$ theory from being near conformal, up to 30% corrections in the case of $N = 3$. Clearly for $N > 3$ our prediction is even more reliable. So we conclude that for $N = 3$ the theory is conformal or quasi-conformal already for $N_f = 2$. We expect this to hold also for the case $N = 2$. The arguments used here about the conformal window are *completely* independent on the nonperturbative methods used above while not disagreeing with our previous results.

The relation with super Yang–Mills can also be employed to deduce vital information about the hadronic spectrum of nonsupersymmetric theories and vice versa [4, 16]. Indeed the $N_f = 1$ bosonic sector of the A/S-type theories, at large N, is mapped into the bosonic spectrum of SYM. In [4] and subsequently in [16] a number of relevant relations for finite N have been uncovered. The $N_f = 1$ sector of a generic N_f theory can then be studied, up to N_f corrections which can be taken into account. Since we are interested in theories with $N_f = 2$ the flavor corrections are small. We recall here some of the results obtained in [4] for the low lying bosonic hadronic states for the A and S type theories with one flavor. These are the generalization of the QCD η' and associated scalar σ, which in SYM are mapped in the pseudoscalar and scalar gluinoballs. In SYM these states are degenerate and have a common mass. Introducing $1/N$ corrections one determines [4]:

$$\frac{M_\sigma}{M_{\eta'}} \simeq 1 \mp \frac{22}{9N} \tag{4}$$

where the upper (lower) sign refers to the S(A) theory. In the previous formula another positive, but numerically small [4], $1/N$ contribution proportional to the gluon condensate has been neglected. The knowledge of the scalar sector of these theories should be confronted with the elusive one in ordinary QCD [17, 18, 19], and consequently in walking as well as in ordinary technicolor theories. If the present theories should ever emerge as the ones driving electroweak symmetry breaking, we would predict that in the S(A) type theory, even near the conformal window, the scalar companion of the techni-η' would be lighter (heavier) than the associated pseudoscalar meson. This suggests that the composite Higgs, which is the chiral partner of the pion and typically lighter than the η'-partner, would also be lighter (heavier) in the S(A) theory than in ordinary walking technicolor type theories. Interestingly, if a light Higgs is discovered it would still be consistent with a scenario of dynamical breaking of the electroweak symmetry and perhaps induce a first order electroweak phase transition in the early universe.

Model Building and Physical Predictions

The minimal model with two fermions, which is almost conformal, has the following S-type matter content and quantum numbers with respect to the Standard Model:

$$\begin{pmatrix} U^{\{c_1,c_2\}} \\ D^{\{c_1,c_2\}} \end{pmatrix}_L \quad \text{with} \quad Y = 0 \ , \tag{5}$$

and $\left(U^{\{c_1,c_2\}}, D^{\{c_1,c_2\}}\right)_R$ with hypercharge $Y = (1, -1)$ and $c_i = 1, \ldots, N$ the gauge indices. This theory has symmetry $SU(2)_L \times SU(2)_R \times U(1)$ for any $N \geq 3$ and it is nearly conformal for $2 \leq N \leq 5$. Let us consider the cases $N = 2$ and $N = 3$ first. The two color theory has a Witten anomaly [20]. To cure such an anomaly without introducing further unwanted gauge anomalies one is forced to introduce at least a new lepton family with half integer charge which should be heavier than the ordinary top quark, but not much heavier. Another interesting point is that the two index symmetric representation of $SU(2)$ is real, and hence the global classical symmetry group is $SU(4)$ which breaks to $O(4)$. This leads to the appearance of nine Goldstone bosons, of which three become the longitudinal components of the weak gauge bosons. Hence, the low energy spectrum is expected to contain six quasi goldstone bosons, which are expected to receive mass through extended technicolor (ETC) interactions [2, 21, 22]. These predictions can be tested at LHC and will be further investigated in much more detail in a near future.

The theory with three technicolors contains an even number of electroweak doublets, and hence it is not subject to a Witten anomaly. Since the two index symmetric representation of $SU(3)$ is complex the flavor symmetry

is $SU(2)_L \times SU(2)_R$. Only three goldstones emerge and are absorbed in the longitudinal components of the weak vector bosons. Based on the discussion reported in the previous section, we predict the Higgs to be lighter than in ordinary walking technicolor theories.

Next we consider the one-family model in our framework. Here we consider A-type technifermions:

$$\left(\begin{array}{c} U^{[c_1,c_2];C} \\ D^{[c_1,c_2];C} \end{array}\right)_L \quad \text{with} \quad Y = y \ , \tag{6}$$

with C the ordinary color index.

$$\left(\begin{array}{c} N^{[c_1,c_2]} \\ E^{[c_1,c_2]} \end{array}\right)_L \quad \text{with} \quad Y = -3\,y \ , \tag{7}$$

and $\left(U^{[c_1,c_2];C}, D^{[c_1,c_2];C}, N^{[c_1,c_2]}, E^{[c_1,c_2]}\right)_R$ with hypercharge $Y = (y+1, y-1, -3y+1, -3y-1)$ and generic y. The charge assignment has been chosen such that the theory is free of gauge anomalies. This theory has symmetry $SU(8)_L \times SU(8)_R \times U(1)$ for any $N \geq 3$ and it is near conformal for $N = 4$ with enhanced symmetry $SU(16)$.

Electroweak precision measurements are an important test for any extension of the standard model. Clearly we need not to worry about the **T** and **U** parameters [23] since our theories naturally have built in custodial symmetry. The **S** parameter whose experimental value is -0.13 ± 0.11 is generally a great problem for technicolor theories. For S(A)-type models the perturbative contribution to the **S** parameter is:

$$\mathbf{S}_{\rm pert.}(S/A) = \frac{1}{6\pi} \cdot \frac{N(N \pm 1)}{2} \cdot \frac{N_f}{2} \ . \tag{8}$$

Near-conformal dynamics leads to a further nonperturbative reduction in the **S** parameter [12]. N_f is the number of techniflavors. It is clear that only a small number of flavors favors a small **S** parameter, making the S-type theories with $N_f = 2$ the best candidates of walking-type dynamics not yet ruled out either at the perturbative or nonperturbative levels [24] by precision measurements. This situation should be confronted with the ordinary walking technicolor dynamics which requires a large number of techniflavors to achieve walking. Clearly this also shows that the A-type model we suggested is not favored by precision measurements as investigated in much detail in the paper [24] based on the present work. We also expect for the A-type theories to have a Higgs heavier than the S-type one and roughly of the same order than the ordinary techicolor type models.

Our findings suggest that, via the same dynamical mechanism, we may be able to explain dynamical symmetry breaking of the electroweak theory, and perhaps even the origin of $SU_L(2)$ as due to an almost conformal theory. However, a more complete theory also able to account for the mass generation of quarks and leptons is needed.

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